ELECTROSTATICS

Note: (*) → Multiple Correct Type Question

1. An isolated metallic object is charged in vacuum to a potential V_0 using a suitable source, its electrostatic energy being W_0. It is then disconnected from the source and immersed in a large volume of dielectric with dielectric constant K. The electrostatic energy of the sphere in the dielectric is: **[NSEP-2017]**

(A) $K^2 W_0$ (B) KW_0 (C) $\frac{W_0}{K^2}$ (D) $\frac{W_0}{K}$

Ans. (D)

Sol. $U=\frac{Q^2}{2C}$ since C will become k times

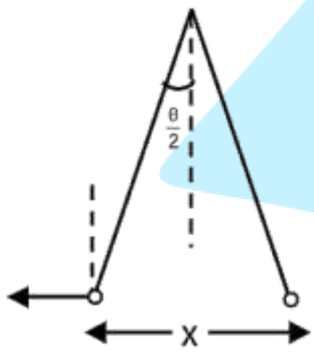
So U will become $\frac{1}{k}$ times

2. Two identical charged spheres suspended from a common point by two light strings of length ℓ, are initially at a distance $d(\ll \ell)$ apart due to their mutual repulsion. The charges begin to leak from both the spheres at a constant rate. As a result, the spheres approach each other with a velocity v. If x denotes the distance between the spheres, the v varies as **[NSEP-2017]**

(A) x^{-1} (B) $x^{1/2}$ (C) $x^{-1/2}$ (D) x

Ans. (C)

Sol.



$$\frac{kq}{x^2}=T\sin\frac{\theta}{2}$$

$$mg=T\cos\frac{\theta}{2} \quad q^2=\frac{mgxx^3}{2k\ell}$$

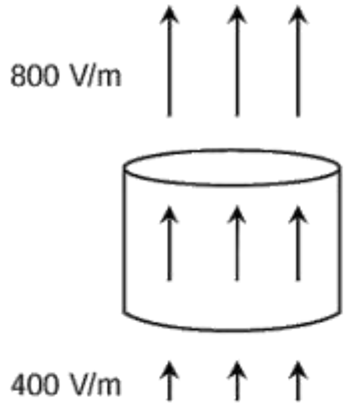
$$\frac{kq^2}{mgx^2}=\tan\frac{\theta}{2} \quad q=\left(\frac{mg}{2k\ell}\right)^{\frac{1}{2}}x^{\frac{3}{2}} \quad q^{\frac{-1}{3}}=\left(\frac{mg}{2k\ell}\right)^{-\frac{1}{6}}x^{-\frac{1}{2}}$$

$$\frac{2kq^2}{mgx^2}=\frac{x}{\ell}\Rightarrow x^3=\frac{2k\ell q^2}{mg}$$

$$2v=\frac{(\text{const})(c)2}{3}q^{-1/3}\Rightarrow x=(\text{ const })q^{\frac{2}{3}}$$

$$2v=2v=(\text{ const })x^{-\frac{1}{2}}\Rightarrow\frac{dx}{dt}=(\text{const})\frac{2}{3}q^{-\frac{1}{3}}$$

3. A cylinder on whose surfaces there is a vertical electric field of varying magnitude as shown. The electric field is uniform on the top surface as well as on the bottom surface therefore, this cylinder encloses **[NSEP-2017]**



(A) no net charge
(B) net positive charge
(C) net negative charge
(D) There is not enough information to determine whether or not there is net charge inside the cylinder.

Ans. (B)

Sol. Consider a Gaussian surface

$\phi = (800-400)A = \frac{q_{in}}{\varepsilon_0}$ $q_{in} = 400_{\varepsilon_0}$ A

4. A hollow conducting sphere of radius 15 cm has a uniform surface charge density $+3.2\mu C/m^2$. When a point charge q is placed at the centre of the sphere, the electric field at 25 cm from the centre just reverses its direction keeping the magnitude the same. Therefore, q is: **[NSEP-2018]**

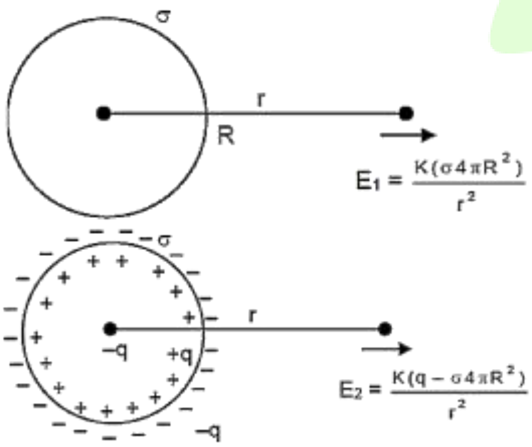
(A) $+0.91\mu C$ (B) $-0.91\mu C$ (C) $+1.81\mu C$ (D) $-1.81\mu C$

Ans. (D)

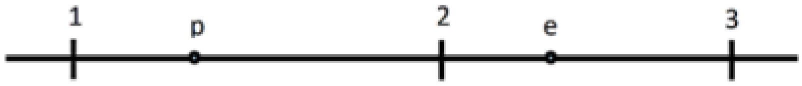
Sol. $E_1 = E_2 \Rightarrow \frac{K\sigma.4\pi R^2}{r^2} = \frac{K}{r^2}(q - \sigma.4\pi R^2) \Rightarrow q = 2 \times 4\pi R^2 \cdot \sigma$

$q = 8\pi R^2\sigma$

$= 1.81\mu c$



5. An electron (e) and a proton (p) are situated on the straight line as shown below. The directions of the electric field at the points 1, 2 and 3 respectively, are shown as **[NSEP-2018]**



(A) $\rightarrow \quad \leftarrow \quad \rightarrow$ (B) $\leftarrow \quad \rightarrow \quad \leftarrow$

(C) $\leftarrow \quad \rightarrow \quad \rightarrow$ (D) $\rightarrow \quad \leftarrow \quad \leftarrow$

Ans. (B)

Sol. $E \propto \frac{1}{r^2}$

*6. Two point charges $+1\mu C$ and $-1\mu C$ are placed at points $(0, -0.1\ m)$ and $(0, +0.1\ m)$ respectively in XY plane. Then, choose the correct statement/s from the following. **[NSEP-2018]**

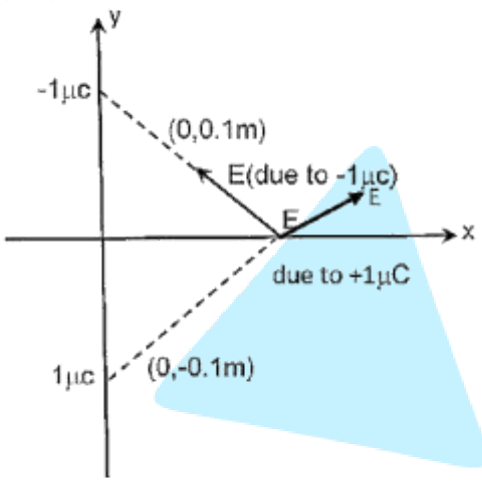
(A) The electric field at all points on the Y axis has the same direction.

(B) The dipole moment is $0.2\mu C - m$ along +X axis direction.

(C) No work has to be done in bringing a test charge from infinity to the origin.

(D) Electric field at all points on the X axis is along +Y axis.

Ans. (CD)



Sol.

$V_0 = 0 \ V\infty = 0 \ \Delta U = q\Delta V = 0 \ \therefore C$ is correct.

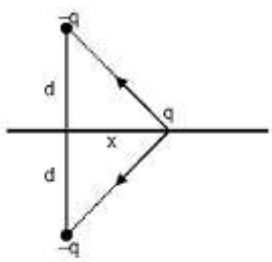
From symmetry of the problem components of field by both charge cancel in x-axis and add up in y-axis.

7. Two charges - q and $-q$ are placed at points $(0, d)$ and $(0, -d)$. A charge $+q$, free to move along X axis, will oscillate with a force proportional to **[NSEP-2019]**

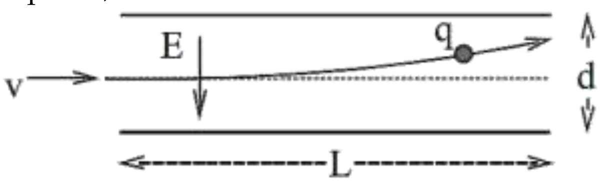
(A) $\frac{1}{x^2+d^2}$ (B) $\frac{1}{x^2}$ (C) $\frac{x}{(d^2+x^2)^{\frac{3}{2}}}$ (D) $\frac{1}{\sqrt{x^2+d^2}}$

Ans. (C)

Sol. $F_{rest} = \frac{2kq^2x}{(d^2+x^2)^{3/2}}$



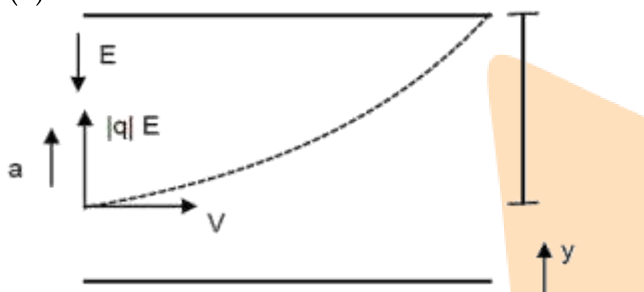
8. In an ink-jet printer, an ink droplet of mass m is given a negative charge q by a computer controlled charging unit. The charged droplet then enters the region between two deflecting parallel plates of length L separated by distance d (see figure below) with a speed v. All over this region there exists a uniform downward electric field E (in the plane of paper). Neglecting the gravitational force on the droplet, the maximum charge that can be given to this droplet, so that it does not hit any of the plates, is: **[NSEP-2019]**



(A) $\frac{mv^2L}{Ed^2}$ (B) $\frac{mv^2d}{EL^2}$ (C) $\frac{md}{Ev^2\ L^2}$ (D) $\frac{mv^2L^2}{Ed}$

Ans. (B)

Sol.



$a = \frac{|q|E}{m}$ A / q when $\Delta x = L \Rightarrow vt = L \Rightarrow t = L/V$

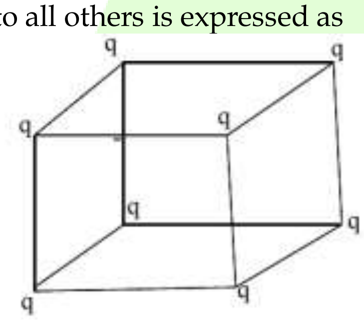
$\Delta y \le d/2 \Rightarrow \frac{1}{2} \times at^2 \le \frac{d}{2} \Rightarrow \frac{1}{2}\frac{|q|Et^2}{m} \le \frac{d}{2} \Rightarrow |q| \le \frac{m\ v^2}{E\ L^2}d$

***9.** An electric dipole placed in a non-uniform electric field may experience **[NSEP-2019]**
(A) No net force, no torque.
(B) A net force, but no torque.
(C) No net force, but a torque.
(D) A net force and a torque.

Ans. (A, B, C, D)

Sol. When a dipole is placed along the electric field at a point where the field is symmetric as well as either maximum or minimum.

10. Point charge q is kept at each corner of a cube of edge length ℓ. The resultant force of repulsion on any one of the charges due to all others is expressed as **[NSEP-2020]**



(A) $\frac{\left(1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}\right)q^2}{\pi\varepsilon_0\ell^2}$ (B) $\frac{\left(\frac{1}{2\sqrt{2}}-1+\frac{1}{3\sqrt{3}}\right)q^2}{\pi\varepsilon_0\ell^2}$

(C) $\frac{(1-0.1775)q^2}{\pi\varepsilon_0\ell^2}$ (D) none of these

Ans. (C)

Sol. The force along x axis is

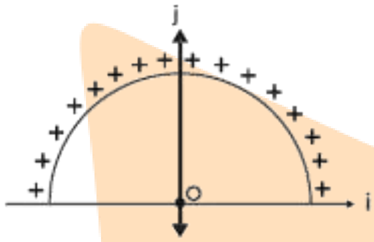
$$F_x = \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{\ell^2} + 2\frac{q^2}{(\ell\sqrt{2})^2}\frac{1}{\sqrt{2}} + \frac{q^2}{(\ell\sqrt{3})^2}\frac{1}{\sqrt{3}}\right] = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\ell^2}\left[1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right]$$

Similar expressions are held along y and z axes. Hence the resultant force is

$$F = \sqrt{F_x^2+F_y^2+F_z^2} \Rightarrow F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\ell^2}\left[1+\frac{1}{\sqrt{2}}+\frac{1}{3\sqrt{3}}\right]\sqrt{\{1+1+1\}} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\ell^2}\left[\sqrt{3}+\sqrt{\frac{3}{2}}+\frac{1}{3}\right] = \frac{0.8225q^2}{\pi\varepsilon_0\ell^2}$$

Or $F = \dfrac{(1-0.1775)q^2}{\pi\varepsilon_0\ell^2}$

11. A thin semi-circular metal ring of radius R has a positive charge q distributed uniformly over its curved length. The resultant electric field $\vec{E}$ at the center O is **[NSEP-2020]**



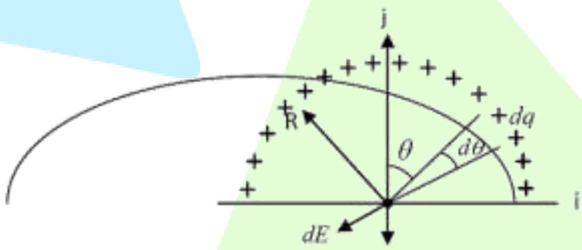
(A) $-\hat{j}\dfrac{q}{2\pi^2\varepsilon_0 R^2}$ (B) $+\hat{j}\dfrac{q}{2\pi^2\varepsilon_0 R^2}$ (C) $+\hat{j}\dfrac{q}{4\pi^2\varepsilon_0 R^2}$ (D) $-\hat{j}\dfrac{q}{4\pi^2\varepsilon_0 R^2}$

Ans. (A)

Sol. By symmetry net electric field along X -axis at the centre O is zero and the electric field along y axis will be added up

$$dE_y = \left(-\hat{j}\right)\frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2}\cos\theta$$

Where $dq = \lambda(Rd\theta) = \dfrac{q}{\pi R}(Rd\theta) = \dfrac{q}{\pi}d\theta$



$$E_y = \left(-\hat{j}\right)2\times\frac{1}{4\pi\varepsilon_0}\int_0^{\frac{\pi}{2}}\frac{\left(\frac{q}{\pi}\cos\theta d\theta\right)}{R^2} \quad E_y = \left(-\hat{j}\right)\frac{q}{2\pi^2\varepsilon_0 R^2}$$

12. A charge $+q$ is placed at each of the points $x = x_0, x = 3x_0, x = 5x_0, x = 7x_0 \ldots\ldots\ldots\ldots\infty$ on the x axis and a charge $-q$ is placed at each of the points $x = 2x_0, x = 4x_0, x = 6x_0, x = 8x_0 \ldots\ldots\infty$ here x_0 is a positive constant. Take the electric potential at a point due to a charge q at a distance r from it to be $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r}$. The electric potential at the origin due to the above system of charges is

[NSEP-2020]

(A) zero (B) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{x_0}\ln 2$ (C) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{x_0 2\ln 2}$ (D) infinite

Ans. (B)

Sol. The potential at the origin may be expressed as

$$V_0 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{x_0} - \frac{q}{2x_0} + \frac{q}{3x_0} - \frac{q}{4x_0} + \frac{q}{5x_0} \ldots\ldots\ldots\right)$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{x_0}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots\ldots\ldots\right)$$

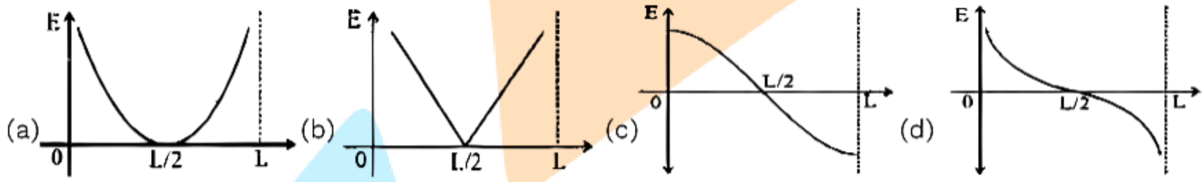
using now

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} \ldots\ldots\ldots \text{ for } -1 < x \le 1$$

$$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} \ldots\ldots\ldots$$

One obtains $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{x_0}\ln 2$

13. Two point charges $+Q$ each are located at $(0,0)$ and $(L,0)$ at a distance L apart on the X - axis. The electric field (E) in the region $0 < x < L$ is best represented by **[NSEP-2021]**



(A) Fig. a (B) Fig. b (C) Fig. c (D) Fig. d

Ans. (D)

Sol. Since both the charges are positive, the electric field at any point between them is

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{x^2} - \frac{Q}{(L-x)^2}\right]$$

This will be positive for $0 < x < \frac{L}{2}$

and negative for $\frac{L}{2} < x < L$ as shown in figure d.

The curve is a $\frac{1}{x^2}$ type.

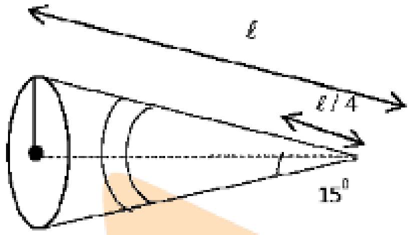
14. A hollow non-conducting cone of base radius $R = 50$ cm and semi vertical angle of 15° has been uniformly charged on its curved surface up to three-fourth of its slant length from base with a surface charge density $\sigma = 2.5\mu C/m^2$. The electric field produced at the location of the vertex of the cone is **[NSEP-2021]**

(A) $\frac{\sigma \ln 2}{2\varepsilon_0}$ (B) $\frac{\sigma \ln 2}{4\varepsilon_0}$ (C) $\frac{\sigma \ln 2}{8\varepsilon_0}$ (D) $\frac{\sigma \ln 2}{16\varepsilon_0}$

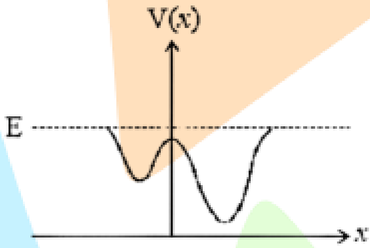
Ans. (B)

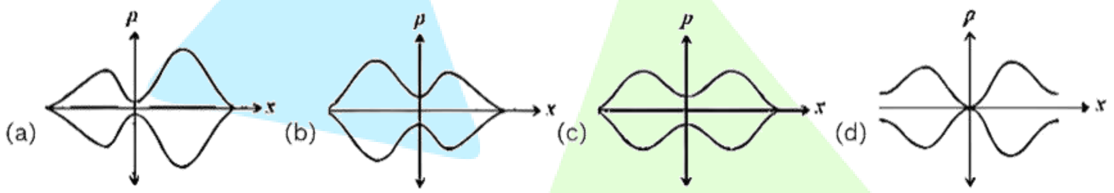
Sol. Let us consider an elementary ring of width $d\xi$ at a slant distance ξ from the vertex of the cone. The charge on the circular ring shall be $dq = 2\pi\xi\sin\theta\, d\xi \times \sigma$. The electric field produced by this elementary ring at the vertex of the cone is $dE = \frac{1}{4\pi\varepsilon_0} \times \frac{2\pi\xi\sin\theta d\xi \times \sigma \times \xi\cos\theta}{\xi^3}$ Thereby the electric field E at the vertex shall be $E = \frac{\sigma}{4\varepsilon_0} 2\sin\theta\cos\theta \int_{1/4}^{1} \frac{d\xi}{\xi} \Rightarrow E = \frac{\sigma}{4\varepsilon_0}\sin 2\theta \{\ln\xi\}_{1/4}^{1}$

$\Rightarrow E = \frac{\sigma}{8\varepsilon_0} \times 2\ln 2 = \frac{\sigma}{4\varepsilon_0}\ln 2$



15. Consider a particle of mass m with a total energy E moving in a one dimensional potential field. The potential $V(x)$ is plotted against x in the figure beside. The plot of momentum - position graph of this particle is qualitatively best represented by **[NSEP-2021]**





All plots are symmetrical about x - axis

(A) Fig. a (B) Fig. b (C) Fig. c (D) Fig. d

Ans. (A)

Sol. Since the total energy is fixed, the kinetic energy so to say the magnitude of the momentum will be large where ever the potential energy is less and vice versa. Further the momentum $p = \pm\sqrt{2m \times (KE)} = \pm\sqrt{2m \times (E - V)}$. Here ($E - V$) is the kinetic energy of the particle. The curve for momentum will be symmetric about x axis so curve a.

In questions 16 and 17 mark your answer as

(A) If statement I is true and statement II is true and also if the statement II is a correct explanation of statement I **[NSEP-2021]**

(B) If statement I is true and statement II is true but the statement II is not a correct explanation of statement I

(C) If statement I is true but the statement II is false

(D) If statement I is false but statement II is true

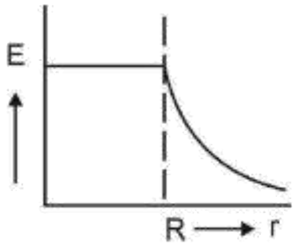
16. Statement I: Work done in bringing a charge q from infinity to the center of a uniformly charged non - conducting solid sphere of radius R (with a total charge Q) is zero.

Statement II: The potential difference between the Centre and the surface of the uniformly charged non - conducting solid sphere of radius R (with a total charge Q) is $\frac{1}{4\pi\varepsilon_0}\times\frac{Q}{2R}$.

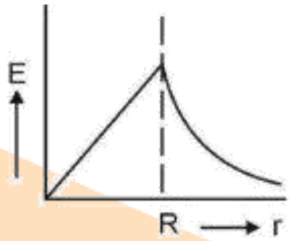
Ans. (D)

Sol. The statement I is false but the statement II is true.

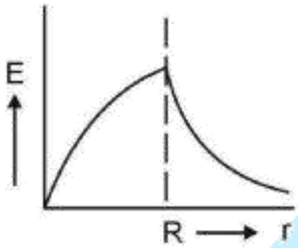
17. A sphere of radius R, is charged with volume charge density ρ such that $\rho \propto r$ (r is distance from Centre). Variation of electric field E with r (For all values of $r : r \le R$ and $r > R$) is best represented by **[NSEP-2022]**



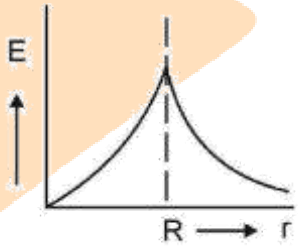
(A)



(B)



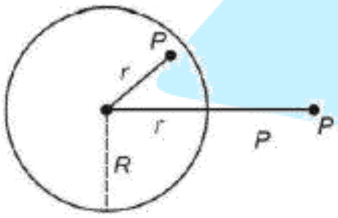
(C)



(D)

Ans. (D)

Sol. For a non-uniformly charged sphere the electric field $E(r)$ is given as



Using Gauss law, $r < R$

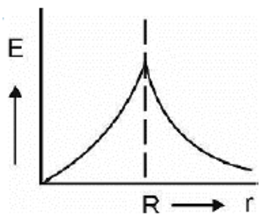
$$\oint \vec{E}.d\vec{A} = \frac{q_{en}}{\varepsilon_0} \Rightarrow E \times 4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^r 4\pi x^2 kx.dx$$

$$E = \frac{kr^2}{4\varepsilon_0}$$

For $r > R$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Therefore, the graph will be drawn as,

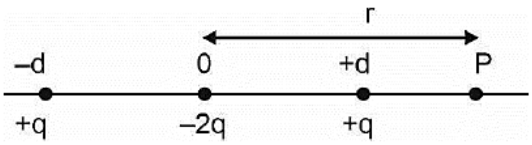


18. Three point charges $+q, -2q$ and $+q$ are placed on x-axis at $x=-d, x=0$ and $x=+d$ respectively. The value of electric field at a point P on x-axis at $x=r(r \gg d)$ is given by $E=\frac{1}{4\pi\varepsilon_0}\frac{aQ}{r^n}$ (Here $Q=2qd^2$). Then **[NSEP-2022]**

(A) $a=3, n=3$ (B) $a=6, n=4$ (C) $a=3, n=4$ (D) $a=\frac{3}{2}, n=4$

Ans. (C)

Sol.



$$E_{net}=\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{(r+d)^2}-\frac{2q}{r^2}+\frac{q}{(r-d)^2}\right)$$

$$=\frac{q}{4\pi\varepsilon_0}\left(\frac{2(r^2+d^2)}{(r^2-d^2)}-\frac{2}{r^2}\right)$$

$$=\frac{2q}{4\pi\varepsilon_0}\left(\frac{r^4+r^2d^2-r^4+2r^2d^2-d^4}{r^2(r^2-d^2)^2}\right)$$

$$=\frac{2q}{4\pi\varepsilon_0}\left(\frac{3r^2d^2-d^4}{r^2(r^2-d^2)^2}\right)$$

$$=\frac{2qd^2}{4\pi\varepsilon_0 r^4}(r \gg d)=\frac{3Q}{4\pi\varepsilon_0 r^4}$$

19. A static point charge Q is located just above the centre $C(\delta 0)$ of a horizontal circle of radius R on its geometric axis, as shown in figure. The magnitude of electric flux through this circle is **[NSEP-2022]**

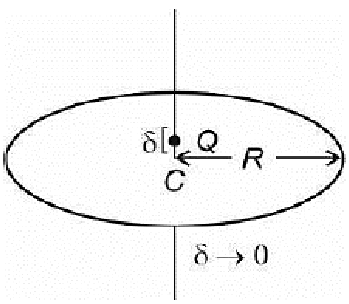
(A) Zero (B) $\frac{Q}{4\varepsilon_0}$ (C) $\frac{Q}{2\varepsilon_0}$ (D) $\frac{Q}{\varepsilon_0}$

Ans. (C)

Sol. Total flux due to $Q=\frac{Q}{\varepsilon_0}$

Flux below the end is almost same as above

$\therefore 2\phi=\frac{Q}{\epsilon_0}\Rightarrow \phi=\frac{Q}{2_{\in 0}}$ $\therefore$ flux through the given circle $=\frac{Q}{2\varepsilon_0}$



20. Three small identical neutral metal balls are at the vertices of an equilateral triangle. The balls are in turn touched to an isolated large charged conducting sphere whose centre is on a line perpendicular to the plane of triangle and passing through its centre. As a result the first and second balls have acquired charges q_1 and q_2 respectively. The charge acquired by the third ball is [Assume that charge and potential of large spherical conductor change insignificantly in charging od the balls and that charges on balls are spherically symmetric] **[NSEP-2022]**

(A) $\frac{q_1^2}{q_2}$ (B) $\frac{q_2^2}{q_1}$ (C) $2q_2-q_1$ (D) $q_3=q_2=q_1$

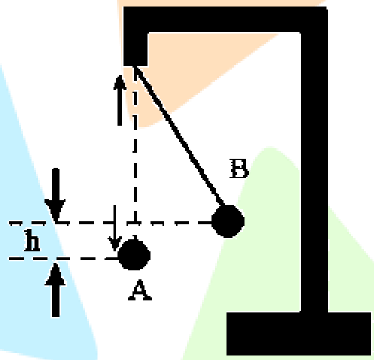
Ans. (B)

Sol. Let $r=$ radius of small ball, R =radius of large sphere, Q =Initial charge of large sphere since on touching, potential would become same

$$\Rightarrow \frac{q_1}{r}=\frac{Q-q_1}{R} \Rightarrow q_1=\frac{r}{R}Q \Rightarrow q_2=\frac{r}{R}\left[1-\frac{r}{R}\right]Q \text{ And } q_3=\frac{r}{R}\left[1-\frac{2r}{R}\right]Q\left[\because r \ll R\right]$$

$$\Rightarrow q_2^2=\left[\frac{r}{R}Q\right]^2\left[1-\frac{2r}{R}\right]=q_1\cdot q_3 \Rightarrow q_3=\frac{q_2^2}{q_1}$$

***21.** A small positively charged ball of mass m is suspended by a long insulating thread of negligible mass. Other positively charged small ball is moved very slowly from a large distance (along horizontal direction) until it is at original position A of first ball. As a result, the first ball rises by h to position B such that $h \ll \ell$. Choose the correct statement(s). **[NSEP-2022]**

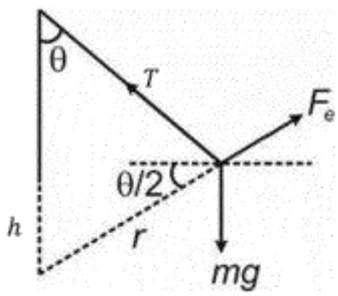


(A) Electrostatic energy of system of charges is 2 mgh .
(B) Total work done on system to bring two balls in their final position is mgh.
(C) Total work done on the system to bring two balls in their final position is 3 mgh .
(D) Work done on system to bring two balls in their final position does not depend on the magnitude of charges explicitly.

Ans. (A, C, D)

Sol. $W_{electrostatics}+W_{gravity}+W_{ex}=\Delta KE$

$W_e-mgh+W_{ex}=0 \quad W_e+W_{ex}=mgh$



Resolving forces as vertical and horizontal components at B,

$$T\sin\theta = F_e\cos\frac{\theta}{2} \Rightarrow F_e = 2T\sin\frac{\theta}{2}$$

$$T\cos\theta + F_e\sin\frac{\theta}{2} = Mg$$

As θ is small, $\cos\theta \approx 1$ and $\sin\frac{\theta}{2}$ is neglected,

$\Rightarrow T = mg$ From the figure, $\sin\sin\frac{\theta}{2} = \frac{h}{r}$

$$\Rightarrow F_e = \frac{2mgh}{r} = \frac{kq_1q_2}{r^2}$$

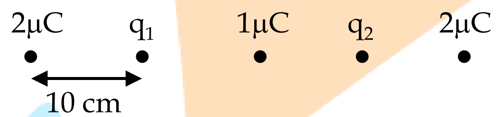
$$\Rightarrow U_e = \frac{kq_1q_2}{r} = 2mgh$$

$$\Rightarrow W_e = -U_e = -2mgh$$

$$\Rightarrow W_{ext} = 3mgh .$$

$\therefore$ Option A, C and D are correct.

22. The figure shows five point-charges on a straight line. Separation between successive charges is 10 cm . For what values of q_1 and q_2 would the net force on each of the other three charges be zero? **[NSEP-2023]**



(A) $q_1 = q_2 = -\frac{27}{80}\mu C$

(B) $q_1 = q_2 = \frac{27}{40}\mu C$

(C) $q_1 = \frac{27}{80}\mu C q_2 = -\frac{27}{80}\mu C$

(D) $q_1 = q_2 = -\frac{27}{40}\mu C, 1\mu C q_2 2\mu C$

Ans. (A)

Sol. $$\frac{K\cdot q_1\times 2}{10^2} + \frac{k\times 1\times 2}{20^2} + \frac{k\times q_2\times 2}{30^2} + \frac{k\times(2)\times(2)}{40^2} = 0$$

$$\frac{q_1}{1} + \frac{1}{4} + \frac{q_2}{9} + \frac{2}{16} = 0$$

$$q_1 + \frac{q_2}{9} + \frac{6}{16} = 0\#(1)$$

$$q_1 + \frac{q_1}{9} = -\frac{6}{16}\ \frac{10q_1}{9} = -\frac{6}{16}\ q_1 = \frac{6\times 9}{160} = -\frac{27}{80}\mu C$$

23. A very small electric dipole of dipole moment $\vec{p}$ lies along the x axis (i.e $\vec{p} = p\hat{i}$) in a nonuniform electric field $\vec{E} = \frac{C}{x}\hat{i}$ (where c is a constant). The force on the dipole is **[NSEP-2023]**

(A) $\frac{CP}{x^2}\hat{i}$ (B) $-\frac{cp}{x^2}\hat{i}$ (C) $\frac{cp}{x}\hat{i}$ (D) zero

Ans. (B)

Sol. $$U = -p\cdot\frac{c}{x}$$

$$f = -\frac{dU}{dx} = -\frac{pc}{x^2}\ \vec{f} == -\frac{c\vec{p}}{x^2}$$

24. A conducting thick spherical shell of radii a and $b(b>a)$ has been charged with uniform surface charge density $-\sigma C/m^2$ on the inner and $+\sigma C/m^2$ on the outer surfaces. Then **[NSEP-2023]**

(A) The net charge on the spherical shell is zero.

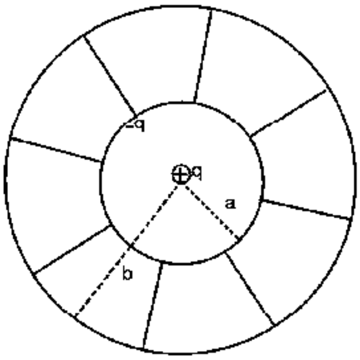
(B) The radial electric field outside the shell is $E=\frac{\sigma b^2}{\varepsilon_0 r^2}$

(C) A Radial electric field $E=\frac{\sigma\left(b^2-a^2\right)}{\varepsilon_0 r^2}$ exists outside the shell.

(D) There is a net electric charge in the cavity (i.e. in region $r<a$) equal to $4\pi\sigma\left(b^2-a^2\right)$

Ans. (B)

Sol.



Total charge $=\sigma.4\pi b^2-\sigma.4\pi a^2=q_1$

$$E=\frac{1}{4\pi\varepsilon_0}\cdot\frac{\sigma\cdot 4\pi b^2}{r^2}=\frac{\sigma b^2}{\varepsilon_0 r^2}$$

25. A spherical conductor is charged up to a potential of 450 V. The potential outside, at a distance 15 cm from the surface, is 300 V. Then **[NSEP-2023]**

(A) The potential at 15 cm from the center is 900 V

(B) The charge on the conductor is 1.5 nC

(C) The electric field just outside the surface is $150\ N/C$

(D) The total electrical energy of the conductor is $U=3.375\mu J$

Ans. (D)

Sol.

$$450=\frac{Kq}{R}$$

$$300=\frac{Kq}{(R+15)}$$

$$\frac{3}{2}=\frac{R+15}{R}$$

$$3R=2R+30$$

$$R=30\ cm$$

$$450=\frac{9\times10^9\times q}{30\times10^{-2}}\Rightarrow q=1500\times10^{-11}$$

$$q=15nc$$

$$E=\frac{Kq}{R^2}=\frac{9\times10^9\times15\times10^{-9}}{(.3)^2}=15\times100=1500$$

$$U=\frac{0.5\times9\times10^9\left(15\times10^{-9}\right)^2}{0.3}=3375\times10^{-9}J=3.375\mu J$$

***26.** A small dipole is placed at the origin with its dipole moment $\vec{P} = p\hat{i}$ oriented along x axis. E and V, are respectively, the Electric field and potential at point $A(x,y)$. The observations at the Point $A(x,y)$ which is at a large distance r from the origin, show that **[NSEP-2023]**

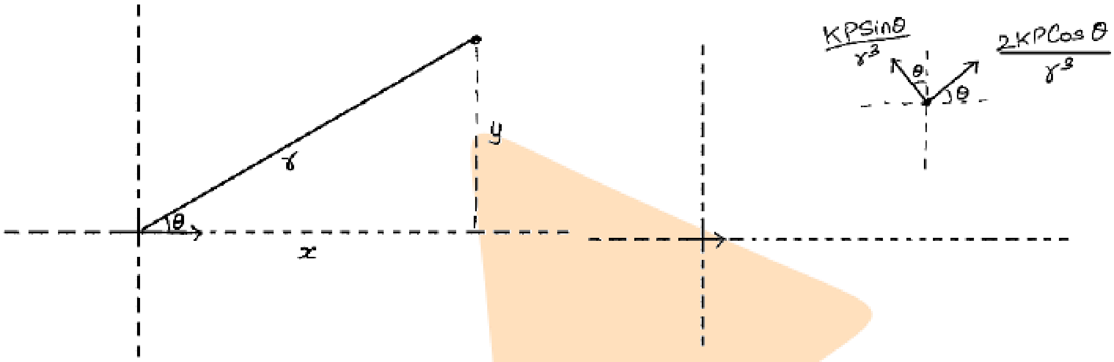
(A) $E_x = \dfrac{1}{4\pi\varepsilon_0}\dfrac{p(2x^2 - y^2)}{r^5}$

(B) $E_x = \dfrac{1}{4\pi\varepsilon_0}\dfrac{p(x^2 - 2y^2)}{r^5}$

(C) $E_y = \dfrac{1}{4\pi\varepsilon_0}\dfrac{3pxy}{r^5}$

(D) $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\vec{p}\cdot\vec{r}}{r^3}$

Ans. (A, C, D)



Sol.

(A) & (B)

$$E_x = \frac{2KpCos\theta}{r^3}\cdot\cos\theta - \frac{Kpsin\theta}{r^3}\cdot\sin\theta$$

$$= \frac{2Kpcos^2\theta}{r^3} - \frac{Kpsin\theta}{r^3}$$

$$= \frac{Kp}{r^3}\left(2\cos^2\theta - \sin^2\theta\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{P}{r^3}\left(2\cdot\frac{x^2}{r^2} - \frac{y^2}{r^2}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{P}{r^5}\left(2x^2 - y^2\right)$$

(C) For Ey

$$E_y = \frac{2Kpcos\theta}{r^3}\cdot\sin\theta + \frac{Kpsin\theta}{r^3}\cdot\cos\theta$$

$$= \frac{3Kpsin\theta cos\theta}{r^3}$$

$$= E_y = \frac{1}{4\pi\varepsilon_0}\frac{3P\cdot xy}{r^5}$$

(D) $V = \dfrac{KPcos\theta}{r^2}$

$$= k\frac{\vec{P}\cdot\vec{r}}{r^3}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{P}\cdot\vec{r}}{r^3}$$

*27. Two equal positive charges $+Q$ each lie on y axis at $(0, a)$ and $(0, -a)$. The electric field strength E at a point $(x, 0)$ satisfies: **[NSEP-2023]**

(A) $E = \frac{1}{4\pi\varepsilon_0}\frac{2Qa}{(x^2+a^2)^{3/2}}$

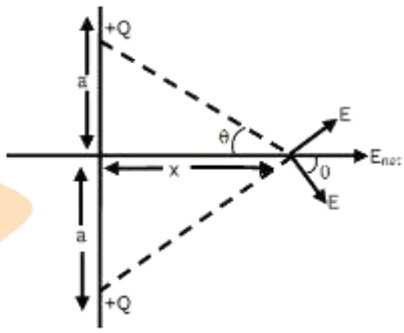
(B) For large values of x (i.e $x \gg a$), the electric field $E \propto \frac{1}{x^2}$

(C) For $x \geq 0, E$ is maximum at $x = \frac{a}{\sqrt{2}}$

(D) For $x \geq 0, E$ is maximum at $x = 0$ and is equal to $\frac{1}{4\pi\varepsilon_0}\frac{2Q}{a^2}$

Ans. (B, C)

Sol. (A) $E_{net} = 2E\cos\theta$



$$= 2\frac{KQ}{a^2+x^2}\cdot\frac{x}{\sqrt{a^2+x^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Qx}{(a^2+x^2)^{3/2}}$$

(B) for $x \ggg a$

$$E_{net} = \frac{2Qx}{4\pi\varepsilon_0 x^3} \propto \frac{1}{x^2}$$

(C) for $x \geq 0, \frac{dE}{dx} = 0$

$$x = \frac{a}{\sqrt{2}}$$

(D) For $x = 0, E = 0$ & not maximum.

GRAVITATION

1. If Newton's inverse square law of gravitation had some dependence on radial distance other than r^{-2}, which on of kepler's three laws of planetary motion would remain unchanged? **[NSEP-2017]**

(A) First law on nature of orbits

(B) Second law on constant areal velocity

(C) Third law on dependence of orbital time period on orbit's semi major axis

(D) None of the above

Ans. (B)

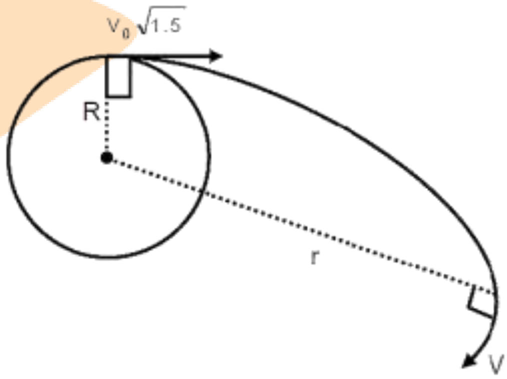
Sol. As the magnitude of force does not matter. The torque would still be zero.

2. A satellite is launched from a point close to the surface of the earth (radius R) with a velocity $v = v_0\sqrt{1.5}$, where v_0 is the velocity in a circular orbit. If the initial velocity imparted to the satellite is horizontal, the maximum distance from the surface of the earth during its revolution is: **[NSEP-2018]**

(A) R (B) 2 R (C) 3 R (D) 4 R

Ans. (B)

Sol. Conservation of angular momentum



$mV_0\sqrt{1.5}R = mVr$

$V = \frac{V_0\sqrt{1.5}R}{r}$ ... (1)

Conservation of energy

$\frac{1}{2}mv_0^2 1.5 - \frac{GMm}{R} = \frac{1}{2}mv^2 - \frac{GMm}{r}$

$\frac{3}{4}\frac{GM}{R} - \frac{GM}{R} = \frac{V_0^2 1.5R^2}{2r^2} - \frac{GM}{r}$

$-\frac{GM}{4R} = \frac{3GMR}{4r^2} - \frac{GM}{r}$

$\frac{3R}{4r^2} - \frac{1}{r} = \frac{-1}{4R}$

$r = R, 3R \Rightarrow$ taking $r = 3R$

distance from surface $= 2R$

3. Let R be the radius of the earth. In general, the loss of gravitational potential energy of a body of mass m falling from a height h to the earth surface is **[NSEP-2018]**

(A) mgh (B) $mgh\frac{R}{R+h}$ (C) $mgh\sqrt{\frac{R+h}{R}}$ (D) $mgh\sqrt{\frac{R}{R+h}}$

Ans. (B)

Sol. $\Delta U = -\frac{GMm}{R} + \frac{GMm}{R+h}$

$\frac{GM}{R^2} = g \Rightarrow GM = gR^2$

$\Delta U = -gmR + \frac{gmRR^2}{R+h} = -gmR\left(1 - \frac{R}{R+h}\right)$

Loss in P.E $= mgh\frac{R}{R+h}$

4. The shortest period of rotation of a planet (considered to be a sphere of uniform density ρ) about its own axis, such that any mass m kept on its equator is just to fly off the surface, is **[NSEP-2020]**

(A) $T=\sqrt{\frac{5\pi}{\rho G}}$ (B) $T=\sqrt{\frac{\pi}{3\rho G}}$ (C) $T=\sqrt{\frac{3\pi}{\rho G}}$ (D) $T=\sqrt{\frac{5\pi}{3\rho G}}$

Ans. (C)

Sol. $g'=g-\omega^2R$ where g' is apparent acceleration due to gravity and $\omega=\frac{2\pi}{T}$ is the angular velocity at the verge of fly off means $g'=0$ or $g=\omega^2R=\left(\frac{2\pi}{T}\right)^2R$ there by

$$\frac{G}{R^2}\frac{4}{3}\pi R^3\rho=\left(\frac{2\pi}{T}\right)^2R\Rightarrow T=\sqrt{\frac{3\pi}{\rho G}}$$

***5.** The magnitudes of the gravitational field at distances r_1 and r_2 from the centre of a uniform solid sphere of radius R and mass M are $F(r_1)$ and $F(r_2)$ respectively. Such that **[NSEP-2020]**

(A) $\frac{F(r_1)}{F(r_2)}=\frac{r_1}{r_2}$ if $r_1\le R$ and $r_2\le R$

(B) $\frac{F(r_1)}{F(r_2)}=\frac{r_2^2}{r_1^2}$ if $r_1\ge R$ and $r_2\ge R$

(C) $\frac{F(r_1)}{F(r_2)}=\frac{r_1}{r_2}$ if $r_1\ge R$ and $r_2\ge R$

(D) $\frac{F(r_1)}{F(r_2)}=\frac{r_1^2}{r_2^2}$ if $r_1\le R$ and $r_2\le R$

Ans. (A, B)

Sol. The gravitational field due to a uniform solid sphere of mass M and radius R at a distance r from its centre is

$F(r)=\left(\frac{GM}{R^3}\right)r$ if $r<R$ and $F(R)=\frac{GM}{r^2}$ if $r>R$ thereby

$\frac{F(r_1)}{F(r_2)}=\frac{r_1}{r_2}$ for $r_1\le R$ and $r_2\le R$ and $\frac{F(r_1)}{F(r_2)}=\frac{r_2^2}{r_1^2}$ for $r_1\ge R$ and $r_2\ge R$

6. Two planets, each of mass M and radius R are positioned (at rest) in space, with their centres a distance 4R apart. You wish to fire a projectile from the surface of one planet to the other. The minimum initial speed for which this may be possible is **[NSEP-2021]**

(A) $\sqrt{\frac{2GM}{5R}}$ (B) $\sqrt{\frac{2GM}{3R}}$ (C) $\sqrt{\frac{4GM}{3R}}$ (D) $\sqrt{\frac{3GM}{2R}}$

Ans. (B)

Sol. Two planets of mass M and radius R each are separated by distance 4 R. A mass m has to be thrown from Planet A so as just to reach Planet B. For this we need to throw the mass so that it just reaches the midpoint then after it will be attracted by B. The potential energy of the mass m on the surface of the planet A is $U_A=-\frac{GMm}{R}-\frac{GMm}{3R}=-\frac{4GMm}{3R}$ and the potential energy at the midpoint between the two planets is $U_{Mid}=-\frac{GMm}{2R}-\frac{GMm}{2R}=-\frac{GMm}{R}$.

Hence the energy needed to project the body is

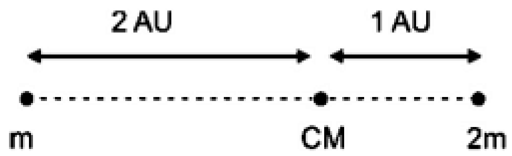
$$\frac{1}{2}mv^2=\left(-\frac{GMm}{R}\right)-\left(-\frac{4GMm}{3R}\right)\Rightarrow v=\sqrt{\frac{2GM}{3R}}$$

7. Two stars of masses M and $m(M=2m)$ separated by a distance $d=3$ astronomical unit, revolve in circular orbit about their centre of mass with a period of 2 years. If M_s is mass of Sun then **[NSEP-2022]**

(A) $m=2.25M_s$ (B) $m=1.25M_s$ (C) $m=2.50M_s$ (D) $m=4.50M_s$

Ans. (A)

Sol. Both the planets are revolving around each other in circular orbit, so $\frac{G2\,m^2}{3^2}=m\omega^2\times 2$



$$\omega=\sqrt{\frac{GM}{9}}\Rightarrow T=2\text{ years }=2\pi\sqrt{\frac{9}{GM}}\Rightarrow\sqrt{GM}=3\pi$$

For Earth, $T=2\pi\sqrt{\frac{1}{GM_s}}=1\text{ year}$

So, $\sqrt{GM_S}=2\pi$

$$\sqrt{Gm}=\frac{3}{2}\sqrt{GM_s}$$

So, $m=\frac{9}{4}M_S=2.25M_S$

8. At the Earth's surface, a projectile is launched straight up at a speed of 10.0 km/s. Height to which it will rise is [gat surface of Earth $=9.8\text{ ms}^{-2}$ and radius of earth $R=6400\text{ km}$] **[NSEP-2022]**

(A) 1.63×10^3 km (B) 2.52×10^4 km (C) 1.56×10^4 km (D) 5.1×10^3 km

Ans. (B)

Sol. Conserving energy:

$$-\frac{GMm}{R}+\frac{1}{2}mv^2=-\frac{GMm}{R+h}+0$$

$$\Rightarrow\frac{-GM}{R+h}=\frac{v^2}{2}-\frac{GM}{R}$$

$$\Rightarrow-\frac{GM}{R^2}\frac{R^2}{R+h}=\frac{v^2}{2}-\frac{GM}{R^2}\cdot R$$

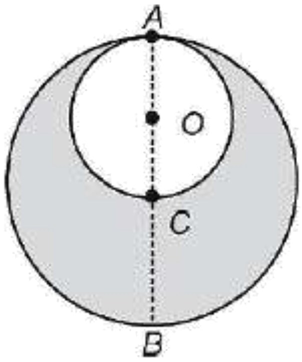
$$\Rightarrow g\left[R-\frac{R^2}{R+h}\right]=\frac{v^2}{2}$$

$$\Rightarrow\frac{Rh}{R+h}=\frac{v^2}{2g}=\frac{\left(10^4\right)^2}{2\times9.8}$$

$$\Rightarrow\frac{h}{R+h}=\frac{10^8}{2\times9.8\times6.4\times10^6}=.797$$

$$\Rightarrow h=\frac{.797R}{0.2}=2.52\times10^4\text{ km}$$

***9.** In an isolated asteroid of radius R and uniform density ρ, a spherical cavity of diameter $AC = R$ is excavated, **[NSEP-2022]**



Where C is centre of asteroid. Choose CORRECT alternative(s)

(A) A ball just dropped from A will strike C with speed $v = 2R\sqrt{\frac{\pi\rho G}{3}}$

(B) A ball dropped from A will reach C after time $t = \sqrt{\frac{3}{\pi\rho G}}$

(C) Acceleration of ball dropped from A varies as its distance from O (centre of cavity)

(D) Weight of a body placed at B (diametrically opposite to A) on the surface of the asteroid decreases by a factor of $\frac{7}{8}$ due to excavation of cavity.

Ans. (A, B)

Sol. Gravitational field inside the gravity is constant and directed parallel to $\vec{OC}$

$\vec{E} = \frac{2}{3}\pi G\rho R\left(-\hat{j}\right)$ So, particle move along AC with constant acceleration

$a = \frac{2}{3}\pi G\rho R$ Using equations of motion for constant acceleration,

$v_c^2 = 2a(AC)$ $v_c^2 = 2\times\frac{2}{3}\pi G\rho R\times R$ $v_c = 2R\sqrt{\frac{\pi\rho G}{3}}$ $\therefore$ option A is correct.

$AC = \frac{1}{2}at^2$ $R = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2R}{a}} = \sqrt{\frac{2R}{\frac{2}{3}\pi G\rho R}} \Rightarrow t = \sqrt{\frac{3}{\pi\rho G}}$ $\therefore$ option B is correct

As acceleration is constant, option C is incorrect.

$E_B = \frac{GM}{R^2}$ (Without excavation)

$$E_B' = \frac{GM}{R^2} - \frac{G\left(\frac{M}{8}\right)}{\left(\frac{3R}{2}\right)^2} = \frac{GM}{R^2} - \frac{4}{9}\frac{GM}{R^2}\cdot\frac{1}{8}$$

$$E_B' = \frac{GM}{R^2} - \frac{GM}{18R^2} = \frac{17}{18}\frac{GM}{R^2}$$

Weight decreases by factor $\frac{17}{18}$

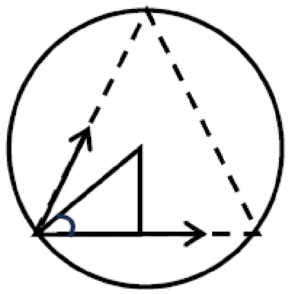
$\therefore$ option D is incorrect.

10. Three stars of equal mass M rotate in a circular path of radius r about their center of mass such that the stars always remain equidistant from each other. The common angular speed (ω) of rotation of the stars can be expressed as **[NSEP-2023]**

(A) $\left(\frac{GM\sqrt{3}}{r^3}\right)^{\frac{1}{2}}$ (B) $\left(\frac{GM}{r^3}\right)^{\frac{1}{2}}$ (C) $\left(\frac{GM}{r^3}\frac{2}{\sqrt{3}}\right)^{\frac{1}{2}}$ (D) $\left(\frac{GM}{r^3\sqrt{3}}\right)^{\frac{1}{2}}$

Ans. (D)

Sol. $a = r\cos(30^\circ) = \frac{\sqrt{3}r}{2}$



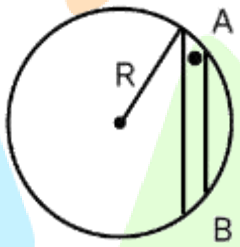
$F_{net} = \sqrt{F^2 + F^2 + 2F^2 \times \frac{1}{2}} = \sqrt{3}F$

$\sqrt{3}F = \frac{\sqrt{3}Gm^2}{(2a)^2} = m\omega^2 r$

$\Rightarrow \frac{\sqrt{3}Gm \times 4}{4 \times \sqrt{3}r^2} = \omega^2 r$

$\Rightarrow \omega = \frac{1}{r}\sqrt{\frac{Gm}{\sqrt{3}}}$

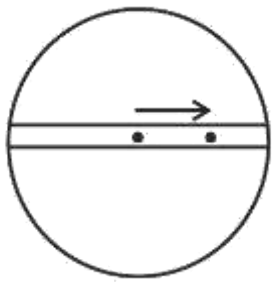
11. The figure shows a smooth tunnel $AB($ length $= 2\ell)$ in a uniform density planet (say Earth) of mass M and radius R. A small ball of mass m is released from rest at the end A of the tunnel. Acceleration due to gravity at surface of the planet is g. Time taken by the ball to reach the end B is **[NSEP-2023]**



(A) $\pi\sqrt{\frac{R}{g}}$ (B) $2\sqrt{\frac{\ell}{g}}$ (C) $\frac{\pi}{2}\sqrt{\frac{2R}{g}}$ (D) $2\pi\sqrt{\frac{R}{g}}$

Ans. (A)

Sol. $\left[\frac{\rho x}{3}\right]4\pi Gm = ma$



$\omega^2 = \left(\frac{4\pi G}{3}\right)\rho \Rightarrow \omega = \sqrt{\left(\frac{4\pi G}{3}\rho\right)}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3}{4\pi G\rho}}$

$\rho = \frac{M}{\frac{4}{3}\pi R^3}$

$T = 2\pi\sqrt{\frac{3 \times 4\pi R^3}{4\pi G \times M \times 3}} = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}}$

Time taken from A to B is $T/2$.